J.D. Jackson Problem 5.27

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April 30, 2012

We'll begin by finding the magnetic field in the three regions. This calculation is somewhat simple because the problem's symmetry allows the use of Ampere's law.

$$\mathbf{B_{in}} = \frac{\mu I r}{2\pi b^2}\hat{\phi} \tag{1}$$

$$\mathbf{B_{mid}} = \frac{\mu_0 I}{2\pi r}\hat{\phi} \tag{2}$$

$$\mathbf{B_{in}} = \mathbf{0} \tag{3}$$

By equation (5.148) and a simplified version of (5.152) we see that

$$L = \frac{1}{I^2}\int \frac{B^2}{\mu}\,\mathrm{d}V \tag{4}$$

$$L = \frac{1}{I^2}\int_0^{2\pi}\int_0^l \left[\int_0^b \frac{\mu_0^2 I^2 r^2}{4\pi^2 \mu b^4} r\,\mathrm{d}r + \int_b^a \frac{\mu_0^2 I^2}{4\pi^2 \mu_0 r^2} r\,\mathrm{d}r + 0\right]\mathrm{d}z\mathrm{d}\theta \tag{5}$$

$$\boxed{\frac{L}{l} = \frac{\mu_0^2}{2\pi}\left[\frac{1}{4\mu} + \frac{1}{\mu_0}\ln\frac{a}{b}\right]} \tag{6}$$

In the case that the inner conductor is hollow, the B-field in the inner-most region becomes zero, and the Inductance per unit length simplifies

$$\boxed{\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{a}{b}} \tag{7}$$

If we return to the more common definition of inductance (**not** per unit length) we get

$$\xi = -\frac{\mathrm{d}}{\mathrm{d}t}\iint \mathbf{B}\cdot\mathrm{d}\mathbf{A} \tag{8}$$

$$= -l\frac{\mathrm{d}}{\mathrm{d}t}\int_b^a \frac{\mu I}{2\pi r}\,\mathrm{d}r \tag{9}$$

$$= -\frac{l\mu_0}{2\pi}\frac{\mathrm{d}I}{\mathrm{d}t}\ln\frac{a}{b} \tag{10}$$

$$\boxed{L = \frac{-\xi}{\frac{\mathrm{d}I}{\mathrm{d}t}} = \frac{l\mu_0}{2\pi}\ln\frac{a}{b}} \tag{11}$$